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Tensor stability in Born-Infeld determinantal gravity

Ke Yang

School of Physical Science and Technology
Southwest University

Based on Ke Yang, Yu-Peng Zhang, Yu-Xiao Liu, arXiv:1812.07348

keyang@swu.edu.cn

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西南大學

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I. Introduction

- Electron's self-energy is divergent in classical electrodynamics.

- Born–Infeld electrodynamics [Proc.R.Soc.London A 144(1934)425]:

$$L_{Newton} = \frac{1}{2}mv^2 \quad \Longrightarrow \quad L_{SR} = -mc^2\left[\sqrt{1-\frac{v^2}{c^2}}-1\right]$$

$$L_{\mathrm{E}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad \Longrightarrow \quad L_{\mathrm{BIE}} = -b^2\left[\sqrt{-\left|\eta_{\mu\nu}-\frac{F_{\mu\nu}}{b}\right|}-1\right]$$

- **General relativity suffers from the singularity problem, too.**

- **Born–Infeld type generalization of gravity**

$$L_{\mathrm{BIG}} = \frac{\lambda}{16\pi G}\left[\sqrt{-\left|g_{\mu\nu} + \frac{2F_{\mu\nu}}{\lambda}\right|} - \Delta\sqrt{-\left|g_{\mu\nu}\right|}\right]$$

where $F_{\mu\nu} = F_{\mu\nu}(\psi, \partial\psi, \cdots)$.

$\lambda \to \infty$

$$L_{\mathrm{BIG}} \approx \frac{1}{16\pi G}\sqrt{-\left|g_{\mu\nu}\right|}\left[\mathrm{Tr}(F_{\mu\nu}) + (1-\Delta)\lambda\right]$$

$$L_{\mathrm{BIG}} \approx \frac{1}{16\pi G}\sqrt{-|g_{\mu\nu}|}\left[\mathrm{Tr}(F_{\mu\nu}) + (1-\Delta)\lambda\right]$$

① $\mathrm{Tr}(F_{\mu\nu}) = R, \; F_{\mu\nu} = R_{\mu\nu}(g)$

$$L_{GR} \approx \frac{\lambda}{16\pi G}\sqrt{-|g_{\mu\nu}|}\left[R + (1-\Delta)\lambda\right]$$

Deser & Gibbons [CQG15(1998)35] **Ghost instability!**

② $\mathrm{Tr}(F_{\mu\nu}) = R, \; F_{\mu\nu} = R_{(\mu\nu)}(\Gamma)$ **Palatini formalism**

Vollick[PRD69(2004)064030]

Banados & Ferrieira [PRL105(2010)011101]

Eddington-inspired Born-Infeld (EiBI) gravity

③ $\mathrm{Tr}(F_{\mu\nu}) = T, \; F_{\mu\nu} = \alpha S_{\mu}{}^{\rho\sigma} T_{\nu\rho\sigma} + \beta S_{\rho\mu}{}^{\sigma} T^{\rho}{}_{\nu\sigma} + \gamma g_{\mu\nu} T, \quad \alpha + \beta + 4\gamma = 1$

Fiorini [PRL111(2013)041104]

$$L_{TG} \approx \frac{\lambda}{16\pi G} e\left[T + (1-\Delta)\lambda\right]$$

Born-Infeld determinantal gravity

II. Eddington-inspired Born-Infeld gravity

$$L_{\mathrm{EiBI}} = \frac{2}{\kappa}\sqrt{-|g_{\mu\nu} + \kappa R_{(\mu\nu)}(\Gamma)|} - \sqrt{-|g_{\mu\nu}|} + L_{\mathrm{matter}}$$

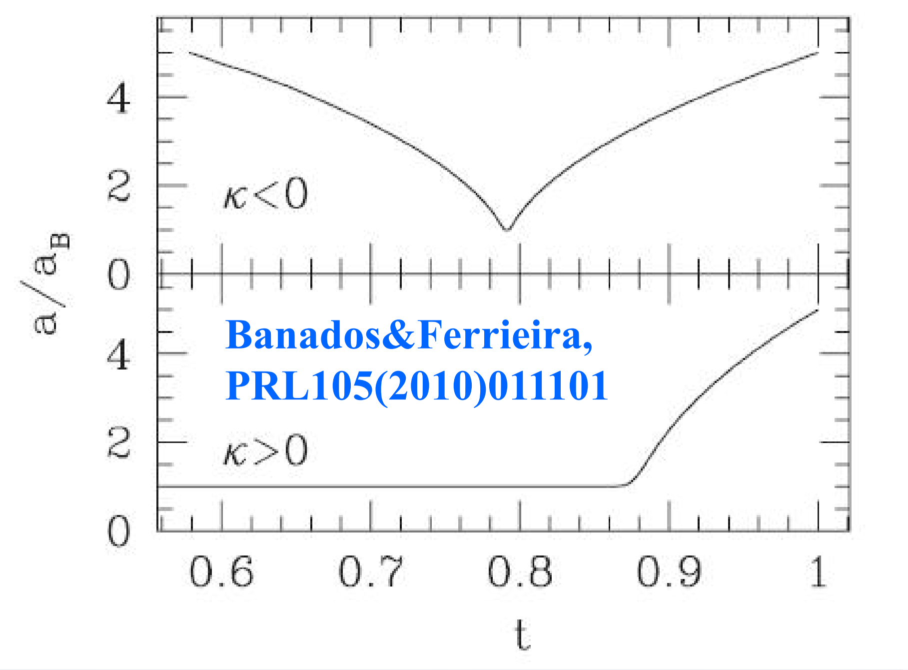
A spatial flat FRW universe with an ideal fluid

$$ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x} \qquad T^{\mu\nu} = Pg^{\mu\nu} + (P+\rho)u^\mu u^\nu$$

Radiation domination $P = \rho/3$:

$\kappa < 0$: $$a = a_B\left(1 - \frac{2}{3\kappa}|t|^2\right)$$

$\kappa > 0$: $$a = a_B\left(1 + e^{\sqrt{\frac{8\kappa}{3}}(t-t_0)}\right)$$



No big bang singularity!

Linear perturbation around the background metric

$$ds^2 = -(1+h_{00})dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j + 2h_{0i}dtdx^i$$

$$\rho = \bar{\rho} + \delta\rho \qquad u^\mu = \bar{u}^\mu + \delta u^\mu$$

Scalar-vector-tensor decomposition:

$$h_{00} = -E$$

$$h_{i0} = \partial_i F + G_i$$

$$\delta u_i = \partial_i \delta u + \delta U_i$$

$$h_{ij} = A\delta_{ij} + \partial_i \partial_j B + \partial_i C_j + \partial_j C_i + D_{ij}$$

where $\partial^i D_{ij} = D^i{}_i = 0$ **, and** $\partial^i C_i = \partial^i G_i = \partial^i \delta U_i = 0$

Modes / κ	Scalar	Vector	Tensor
$\kappa > 0$	Unstable	Stable	Unstable
$\kappa < 0$	Unstable	Unstable	Unstable

K. Yang, X.-L. Du, Y.-X. Liu, PRD88(2013)124037

C. Escamilla-Rivera, M. Banados, P.G. Ferreira, PRD85(2012) 087302

III. Born-Infeld determinantal gravity

$$L_{\mathrm{BIDG}} = \frac{\lambda}{16\pi G}\sqrt{-\left|g_{\mu\nu} + \frac{2}{\lambda}F_{\mu\nu}\right|} - \sqrt{-|g_{\mu\nu}|} + L_{\mathrm{matter}}$$

where $F_{\mu\nu} = \alpha S_{\mu}{}^{\rho\sigma}T_{\nu\rho\sigma} + \beta S_{\rho\mu}{}^{\sigma}T^{\rho}{}_{\nu\sigma} + \gamma g_{\mu\nu}T$ and $\alpha + \beta + 4\gamma = 1$.

Equation of motion [Fiorini & Vattuone, PLB763(2016)45]:

$$\frac{|U_{\mu\nu}|^{\frac{1}{2}}(U^{-1})^{\beta\alpha}}{2}\left[\delta^{\mu}{}_{(\alpha}g_{\nu\beta)} + \frac{2e^{A}{}_{\nu}}{\lambda}\frac{\partial F_{\alpha\beta}}{\partial e^{A}{}_{\mu}}\right] - \frac{e^{A}{}_{\nu}}{\lambda}\partial_{\gamma}\left[|U_{\mu\nu}|^{\frac{1}{2}}(U^{-1})^{\beta\alpha}\frac{\partial F_{\alpha\beta}}{\partial(\partial_{\gamma}e^{A}{}_{\mu})}\right] - \delta^{\mu}{}_{\nu}e = \frac{16\pi G e}{\lambda}\Theta_{\nu}{}^{\mu}$$

where $U_{\mu\nu} \equiv g_{\mu\nu} + 2\lambda^{-1}F_{\mu\nu}$ and $\Theta_{\nu}{}^{\mu} = -e^{-1}\partial L_{M}/\partial e^{A}{}_{\mu}$.

For an ideal fluid: $\Theta_{\nu}{}^{\mu} = (P + \rho)u^{\mu}u_{\nu} - P\delta^{\mu}{}_{\nu}$

A spatial flat FRW universe:

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j$$

The corresponding vierbein reads:

$$e^A{}_\mu = \begin{pmatrix} 1 & 0 \\ 0 & a(t)\delta^a{}_i \end{pmatrix}$$

Equations of motion:

$$\frac{\sqrt{1 - BH^2}}{\sqrt{1 - AH^2}}(1 + 2BH^2 - 3ABH^4) - 1 = \frac{16\pi G}{\lambda}\rho$$

$$\frac{\sqrt{(1 - BH^2)^{-1}}}{\sqrt{(1 - AH^2)^3}}\left[1 + \frac{A + 3B}{3}\dot{H} - (A - B)H^2 - \frac{14AB + 6B^2}{3}H^2\dot{H} - (4AB + 2B^2)H^4 \right.$$
$$\left. + \frac{(9A + 19B)AB}{3}H^4\dot{H} + (3A + 5B)ABH^6 - 4A^2B^2H^6\dot{H} - 3A^2B^2H^8\right] - 1 = -\frac{16\pi G}{\lambda}\rho$$

where $A = 6(\beta + 2\gamma)/\lambda$ **and** $B = 2(2\alpha + \beta + 6\gamma)/\lambda$.

Perturbed metric:

$$ds^2 = dt^2 - a^2(t)[\delta_{ij} + 2h_{ij}(t,\vec{x})]dx^i dx^j$$

where $\partial^i h_{ij} = h^i{}_i = 0.$

The corresponding perturbed vierbein:

$$e^A{}_\mu = \begin{pmatrix} 1 & 0 \\ 0 & a(t)(\delta^a{}_i + \delta^a{}_j h^j{}_i) \end{pmatrix}$$

Substitute the perturbed vierbein into the equation of motion:

$$\frac{|U_{\mu\nu}|^{\frac{1}{2}}(U^{-1})^{\beta\alpha}}{2}\left[\delta^\mu{}_{(\alpha} g_{\nu\beta)} + \frac{2e^A{}_\nu}{\lambda}\frac{\partial F_{\alpha\beta}}{\partial e^A{}_\mu}\right] - \frac{e^A{}_\nu}{\lambda}\partial_\gamma\left[|U_{\mu\nu}|^{\frac{1}{2}}(U^{-1})^{\beta\alpha}\frac{\partial F_{\alpha\beta}}{\partial(\partial_\gamma e^A{}_\mu)}\right] - \delta^\mu{}_\nu e = \frac{16\pi G e}{\lambda}\Theta_\nu{}^\mu$$

Evolution equation: $F_2 \ddot{h}^i{}_j + F_1 \dot{h}^i{}_j + F_0 k^2 h^i{}_j = 0$

$$F_0 = \frac{4}{3a^2}\left[\frac{D}{1-AH^2} + \frac{3-D\lambda}{\lambda(1-BH^2)}\right]$$

$$F_1 = \frac{AH}{1-AH^2} + \frac{4(B-D)H}{1-BH^2} - \frac{A^2 H\dot{H}}{3(1-AH^2)^2} - \frac{2ABH\dot{H}}{(1-AH^2)(1-BH^2)} - \frac{BC^2H^3\dot{H}}{(1-BH^2)^3} + \frac{AC^2H^3\dot{H}}{(1-AH^2)(1-BH^2)^2} - \frac{4CH + 2(B^2+2C^2)H\dot{H} - (3B-4C+4D)CH^3}{2(1-BH^2)^2}$$

$$F_2 = \frac{A}{3(1-AH^2)} + \frac{B}{1-BH^2} - \frac{C^2H^2}{(1-BH^2)^2}$$

where $C = (2\alpha+\beta)/\lambda$ **and** $D = 3\gamma/\lambda$.

In low energy regime ($\lambda \to \infty$ **):** $\ddot{h}^i{}_j + 3H\dot{h}^i{}_j + \frac{k^2}{a^2} h^i{}_j = 0$

It reduces to the standard evolution equation in GR.

- **Solution I [Fiorini, PRL111(2013)041104]**

$B = 0$ ⇒ $\beta = \alpha + 3$ $\gamma = -(1+\alpha)/2$

$$\left(1 - \frac{12H^2}{\lambda}\right)^{-\frac{1}{2}} - 1 = \frac{16\pi G}{\lambda}\rho$$

$$\left(1 - \frac{16H^2}{\lambda} + \frac{4a''}{\lambda a}\right)\left(1 - \frac{12H^2}{\lambda}\right)^{-\frac{3}{2}} - 1 = -\frac{16\pi G}{\lambda}P$$

In the asymptotic past $t \to -\infty$, the scale factor behaves like

$$a(t) \propto e^{\sqrt{\frac{\lambda}{12}}t}\left(1 + O\left(e^{\sqrt{3\lambda}(1+\omega)t}\right)\right)$$

- **It possesses a geometrical de Sitter inflationary stage.**
- **The Hubble factor approaches a maximum value $H(t \to -\infty) = \sqrt{\lambda/12}$**
- **For every state parameter $\omega > -1$, the solution describes a geodesically complete spacetime without the big bang singularity.**

Evolution equation: $$\ddot{h}^i{}_j + \frac{F_1}{F_2}\dot{h}^i{}_j + \frac{F_0}{F_2}k^2 h^i{}_j = 0$$

$$\frac{F_1}{F_2} \approx -\frac{\sqrt{3\lambda}}{2}\omega + \mathcal{O}(e^{\sqrt{3\lambda}(1+\omega)t})$$

$$\frac{F_0}{F_2} \approx -\frac{e^{-\sqrt{\frac{\lambda}{3}}t}}{8}\left[4(1+\alpha) + \mathcal{O}\left(e^{\sqrt{3\lambda}(1+\omega)t}\right)\right]$$

$\alpha < -1$	**Stable:** $\omega \geq -\frac{1}{3}$	**Unstable:** $-1 < \omega < -\frac{1}{3}$
$\alpha = -1$	**Stable:** $\omega > 0$	**Unstable:** $-1 < \omega \leq 0$
$\alpha > -1$	**Unstable**	

- **Solution II [Fiorini, PRD94(2016) 024030]**

$$A = B \Rightarrow \alpha = \beta = \frac{1}{2} - 2\gamma$$

$$3H^2\left(1 - \frac{9H^2}{2\lambda}\right) = 8\pi G\rho$$

$$3H^2\left(1 - \frac{9H^2}{2\lambda}\right) + 2\dot{H}\left(1 - \frac{9H^2}{\lambda}\right) = -8\pi GP$$

For $\lambda > 0$ and $\omega > -1$, a brusque bounce solution can be obtained, and around the bounce point ($t = 0$), the scale factor behaves like

$$\frac{a(t)}{a_0} \approx \left(\frac{\rho_0}{\rho_m}\right)^{\frac{1}{3(1+\omega)}}\left[1 \pm \frac{\sqrt{\lambda}}{3}t - \frac{\pm\lambda^{\frac{3}{4}}(1+\omega)^{\frac{1}{2}}}{9}(\pm t)^{\frac{3}{2}}\right] + \mathcal{O}(t^2)$$

where the positive (minus) sign corresponds to $t > 0$ ($t < 0$).

- **Hubble factor approaches a maximum value $H(0) = \pm\sqrt{\lambda}/3$**
- **The the cosmic time derivative of the Hubble rate reads $\dot{H} \propto \frac{1}{\sqrt{|t|}}$**

Evolution equation: $F_2 \ddot{h}^i{}_j + F_1 \dot{h}^i{}_j + F_0 k^2 h^i{}_j = 0$

$$F_2 \approx -\frac{3K_2}{16\lambda} \pm \frac{3(1+\omega)^{\frac{1}{2}}(24+K_2)}{8\lambda^{\frac{3}{4}}}\sqrt{\pm t} + \mathcal{O}(t)$$

$$F_1 \approx \frac{3K_1(1+\omega)^{\frac{1}{2}}}{32\lambda^{\frac{3}{4}}\sqrt{\pm t}} \pm \frac{144-(5+2\omega)K_1}{16\sqrt{\lambda}} + \mathcal{O}(t^{\frac{1}{2}})$$

$$F_0 \approx \frac{6}{a_0^2\lambda}\left(\frac{\rho_m}{\rho_0}\right)^{\frac{2}{3(1+\omega)}} + \mathcal{O}(t^{\frac{1}{2}})$$

where $K_1 = 48\gamma^2 - 24\gamma + 19$ **and** $K_2 = 48\gamma^2 - 24\gamma - 29$.

- **Stable for $\omega > -1$ and any γ.**

IV. Summary

- For solution I, the tensor evolution is stable for $\alpha < -1, \omega > -\frac{1}{3}$ and $\alpha = -1, \omega > 0$.

- For solution II, the tensor evolution is stable for $\omega > -1$ and any γ.

- The cosmic evolution against tensor perturbation is unstable in EiBI gravity, but stable in large parameter spaces in Born-Infeld determinantal gravity, which is a remarkable property.

Thank you!

keyang@swu.edu.cn